FINANCIAL ECONOMICS WORKSHOP 2010

Bayesian Techniques in Portfolio Management

Bell Associates LLC

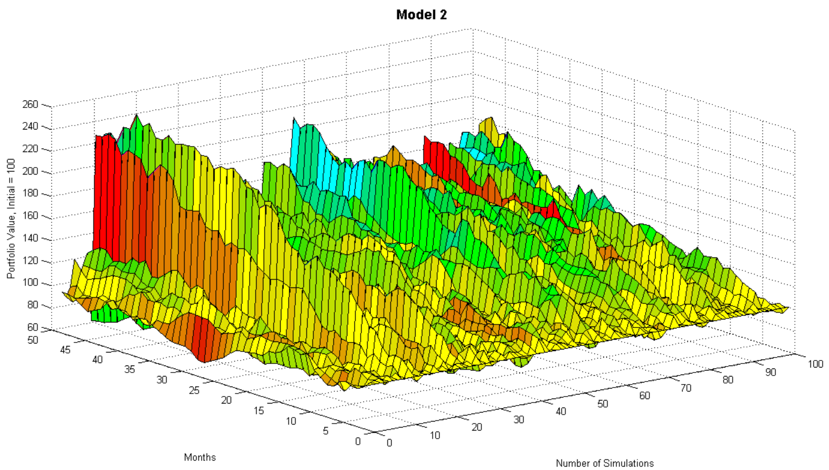
May 12, 2010

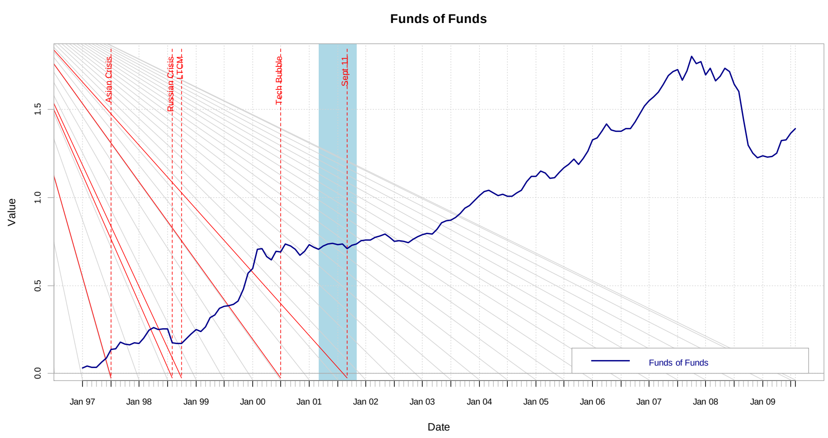
Quant Finance Seminar 2010

- Estimation
- Mean Variance
- Bayesian Techniques
- Estimating Volatility

ESTIMATING MARKET INVARIANTS

Asset Class Modeling





Equities

Fixed Income

Commodities

Currencies

Alternatives

Exotic Beta (e.g. fine art)

Market Invariants

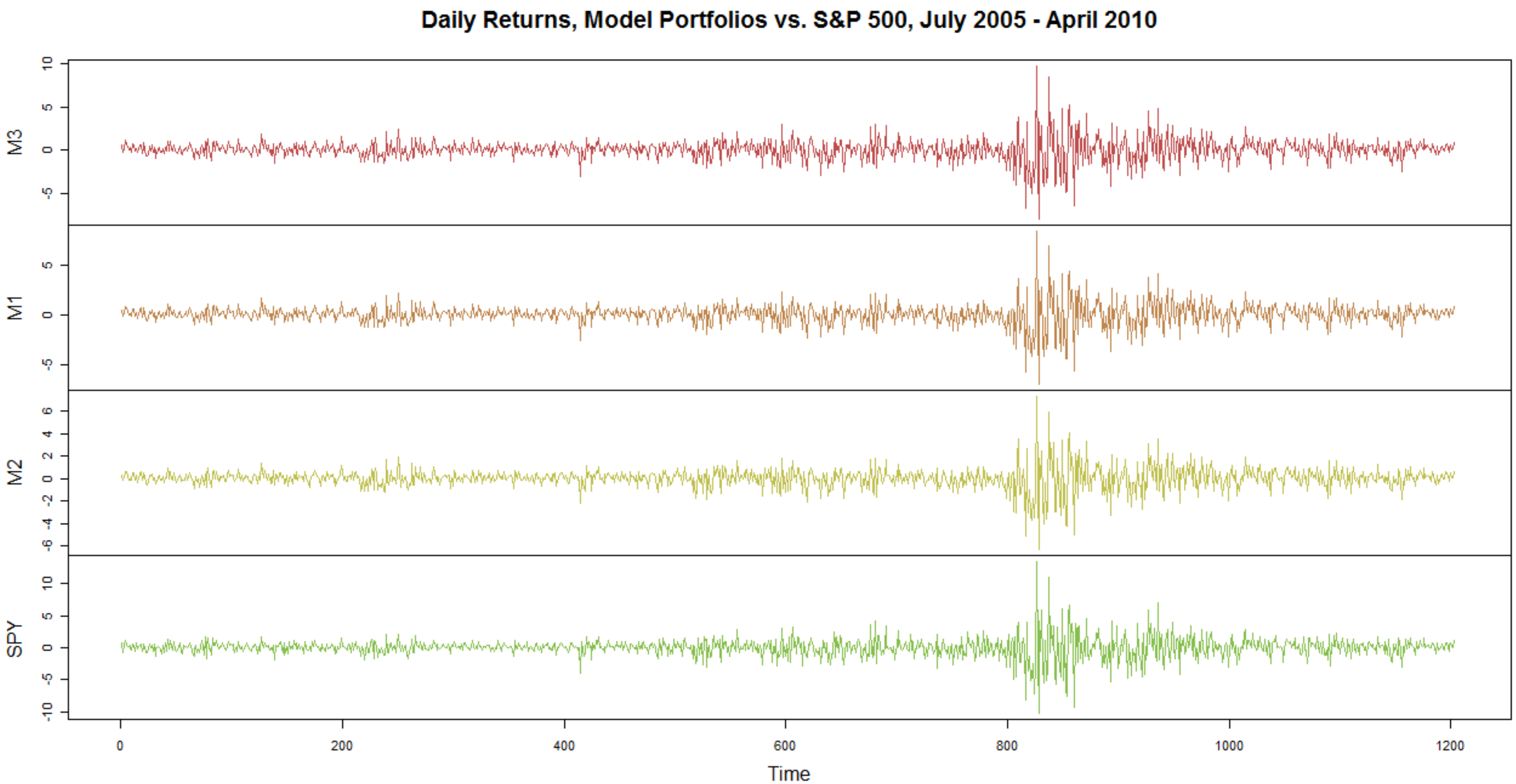
The basic Idea is to look for independent, identically distributed random variables from the market data.

- Linear returns and log-returns in the equity market
- Changes in yield-to-maturity in the fixed income market
- Changes in implied volatility in the options market (at the money forward or ATMF)

Estimation Process

1. Look for market invariants
2. Fit a distribution to the data (e.g. normal distribution, lognormal, Student's t, Cauchy, inverse Wishart etc.)
3. Project their distribution to the investment horizon
4. Transform the projected market invariant distribution into a market price distribution (e.g. from returns to a horizon price distribution for equities)

Returns Time Series



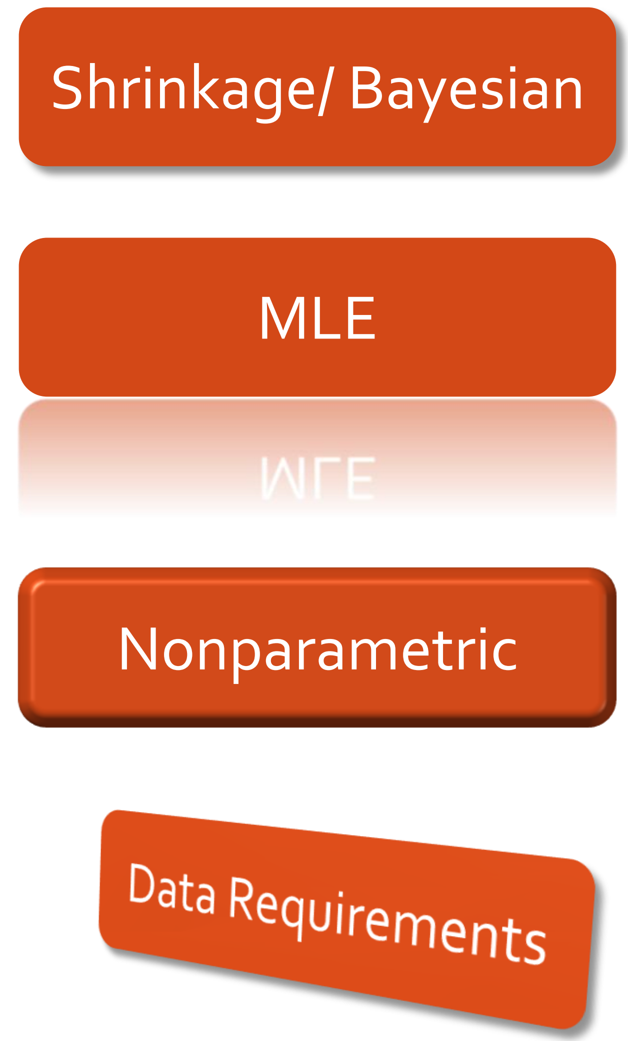
Estimation

Depending on the amount of information available:

•Shrinkage or Bayesian estimates (very little data)

•Maximum likelihood estimators (MLE) (more data)

•Nonparametric estimators (a lot of data)



Estimation (Continued)

Using long data samples generally helps to reduce estimation risk but:

Long data samples are not always available (e.g. developing markets, new ETFs, alternatives).

Distributions over long periods of time can be time varying.

Since more recent observations contain less stale information, they can be given greater weight (i.e. exponential weighting schemes).

Shrinkage/ Bayesian

MLE

Nonparametric

Data Requirements

Estimation (Continued)

The quality of an estimator can be measured by its error (mean square error), its bias (average distance to the correct value) and its inefficiency (standard deviation, i.e., dispersion around its expected value):

$\text{Error}^2 = \text{Bias}^2 + \text{Inefficiency}^2$

Error can be decomposed into two parts:

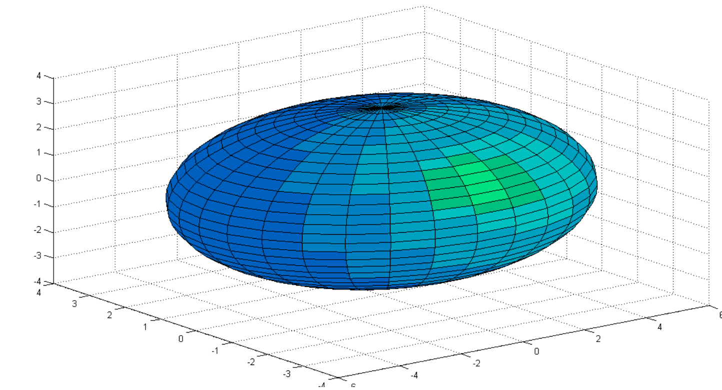
Bias

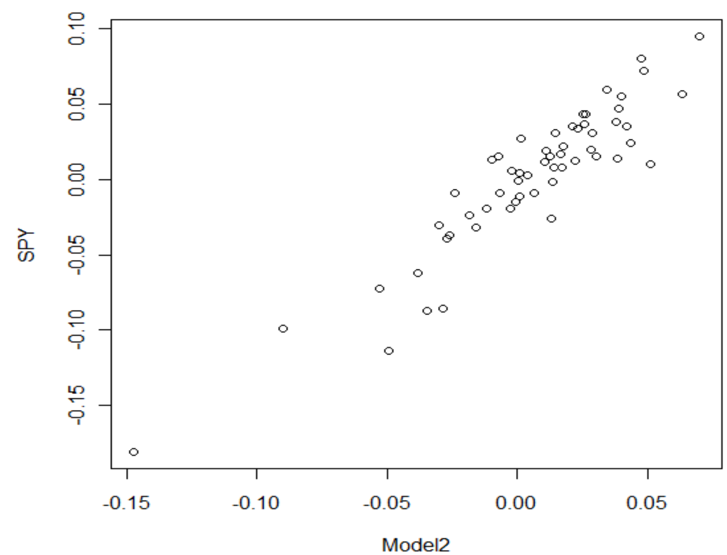
Inefficiency

Outlier Detection

The minimum volume ellipsoid (MVE) finds the smallest ellipsoid containing x% of the data for various values of x.

The minimum covariance determinant (MCD) is the minimum value of the determinant of the same covariance matrix containing x% of the data.





MEAN VARIANCE

Mean Variance Optimization

"Classical" mean-variance optimization (MVO) is overly sensitive to errors in inputs (means and covariances of returns) as sample moments are considered to be the true moments.

Robust Mean Variance

- Developed by Olivier Ledoit and colleagues on the statistical arbitrage trading desk at Credit Suisse London and the University of Zurich, 2002-2005
- Uses the MCD outlier detection algorithm and the Ledoit-Wolf shrinkage estimator to improve estimator performance
- In practice, one shrinks sample data to a fixed target- typically chosen through a factor-based model.

Robust Mean Variance

Variations:

- Kendall’s rank estimator
- Spearman's rank estimator
- Minimum covariance determinant estimator
- Minimum volume ellipsoid estimator
- Orthogonalized Gnanadesikan-Kettenring
- Shrinkage estimator
- Nearest neighbor variance estimator
- Covariance sample estimator (Basic Mean-Variance)

Investor Objectives

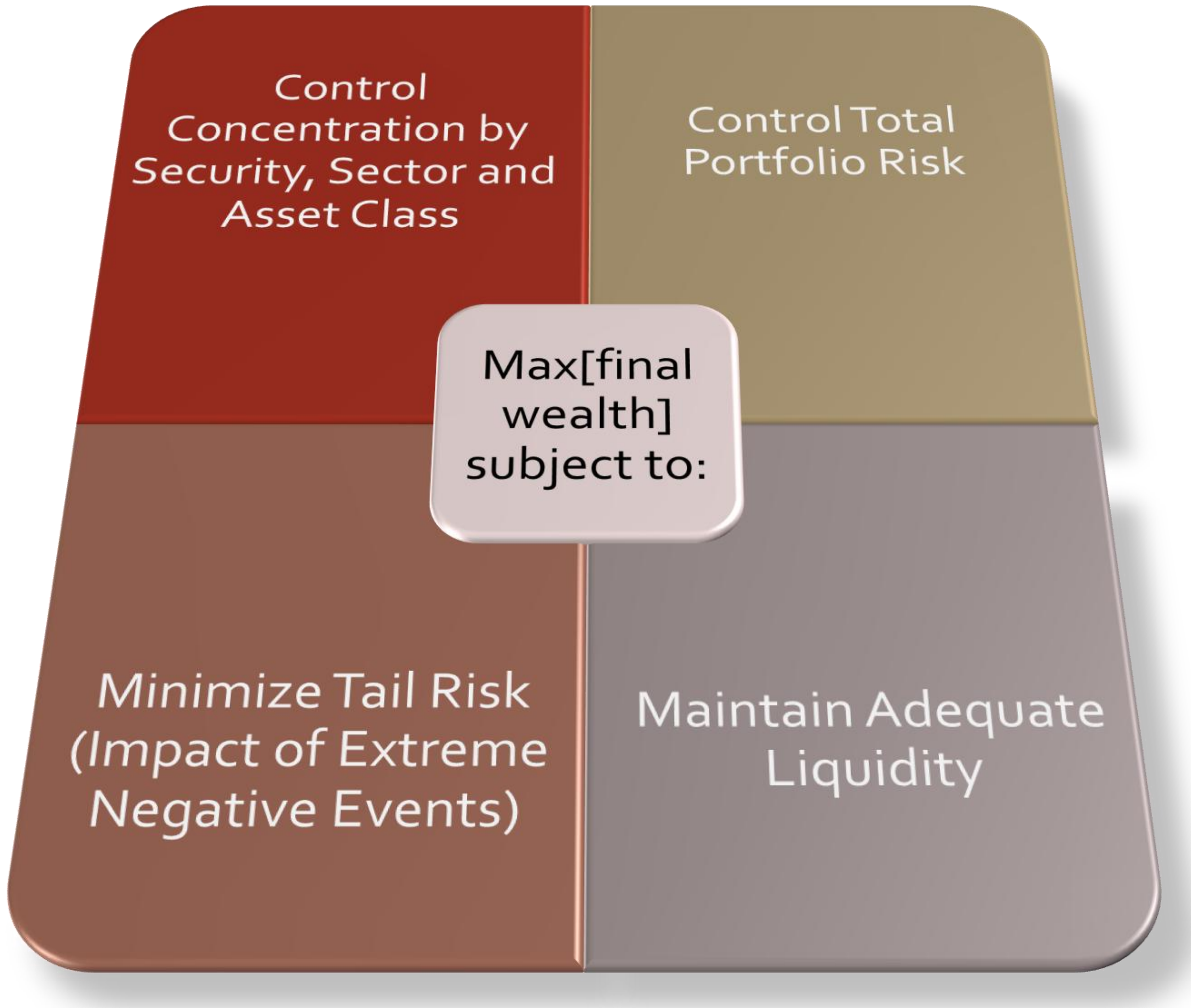
- Money-equivalence (units in money)
- Sensibility
- Constancy
- Risk Aversion

Technical:

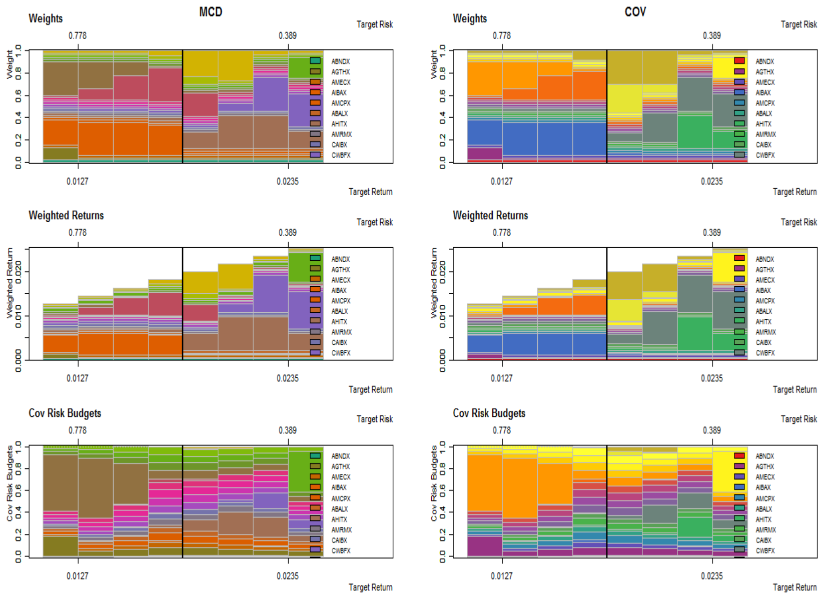
- Positive homogeneity
- Translation invariance
- Super-additivity
- Comonotonic additivity
- Concavity

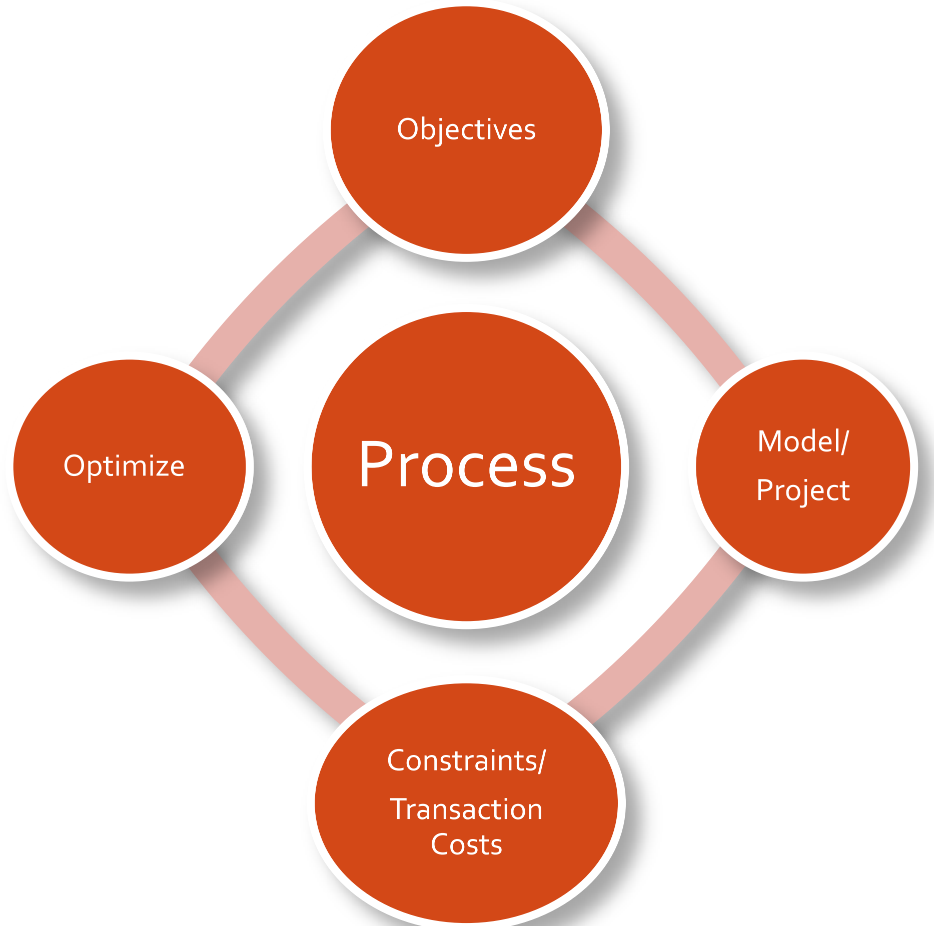
Examples

- Expected Value of the Portfolio
- Sharpe Ratio
- Certainty-equivalent
- Value at Risk
- Expected Shortfall


Control Concentration by Security, Sector and Asset Class
Control Total Portfolio Risk
Max[final wealth] subject to:
Minimize Tail Risk (Impact of Extreme Negative Events)
Maintain Adequate Liquidity

Process Map (Clockwise)





Portfolio Construction

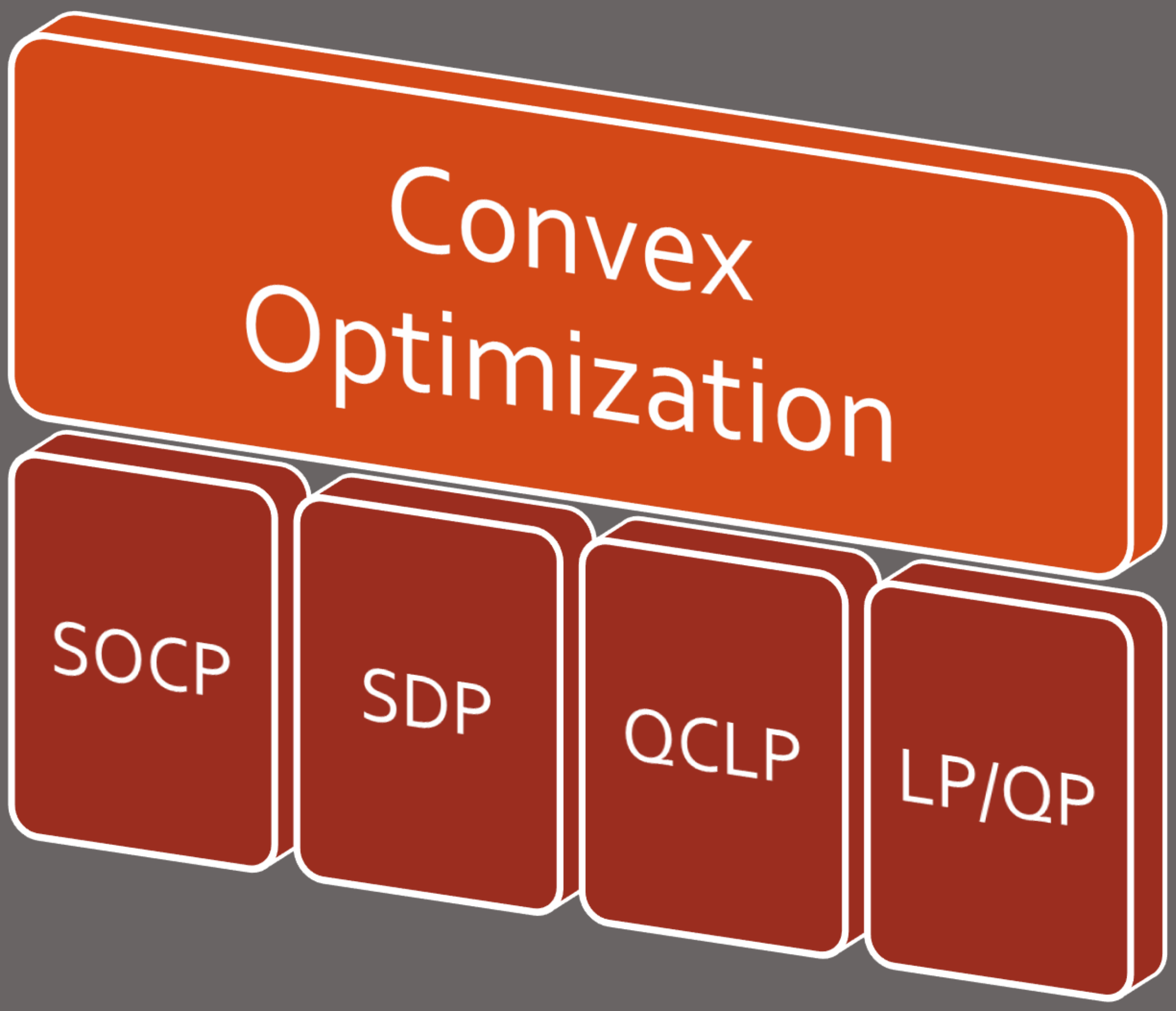
Process Map:

1. Define the investor’s objective
2. Model the market invariants
3. Project the market invariants to the investment horizon
4. Gather other information: constraints, transaction costs, taxes
5. Maximize the investor’s objective subject to constraints

Optimization Techniques

Convex Programming/Interior Point Methods:

- Linear programming (LP)
- Quadratic programming (QP)
- Quadratically-constrained linear programming (QCLP)
- Second-order cone programming (SOCP)
- Semi-definite programming (SDP)


Convex Optimization
SOCP
SDP
QCLP
LP/QP

BAYESIAN ALLOCATIONS

Bayesian Approach

- Estimation risk is accounted for by considering parameters as random variables.
- A prominent example is the Black-Litterman model (see next).
- Model conclusions blend the information content of observed data with subjective information (e.g. Markov Chain Monte Carlo techniques).
- Instead of a single point estimate (e.g. sample mean), one obtains a distribution for the parameter.

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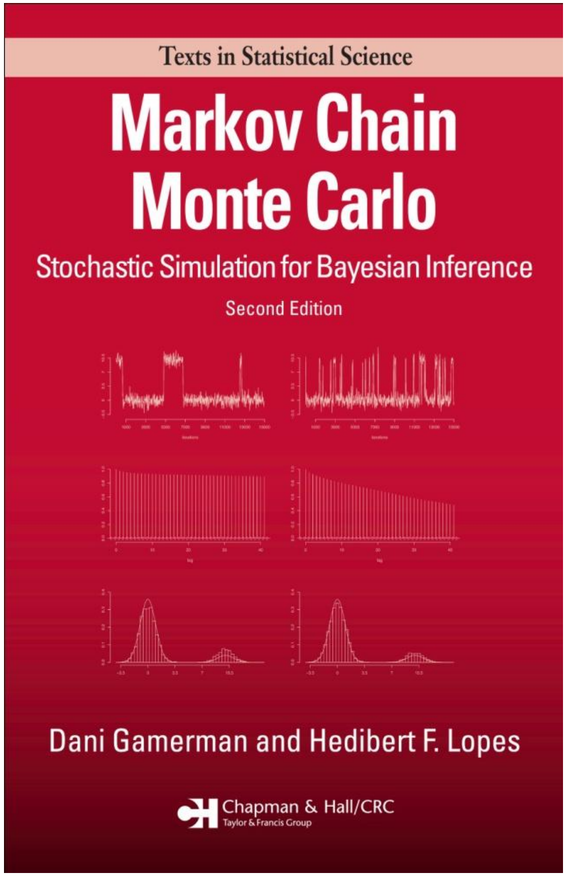
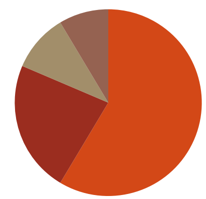


Relative Value Views by Security or Asset Class

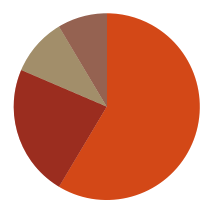
As a toy example, consider a portfolio of a bond and 3 risky stocks.

Investors specify views on the expected returns of as few assets as they choose.

Expected returns of assets with no views are centered on equilibrium expected returns (as determined by an equilibrium pricing model such as the CAPM).



1	2	3
1	2	3
1	2	3



1	2	3
1	2	3
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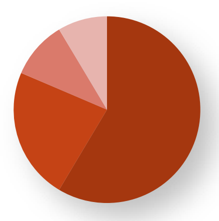


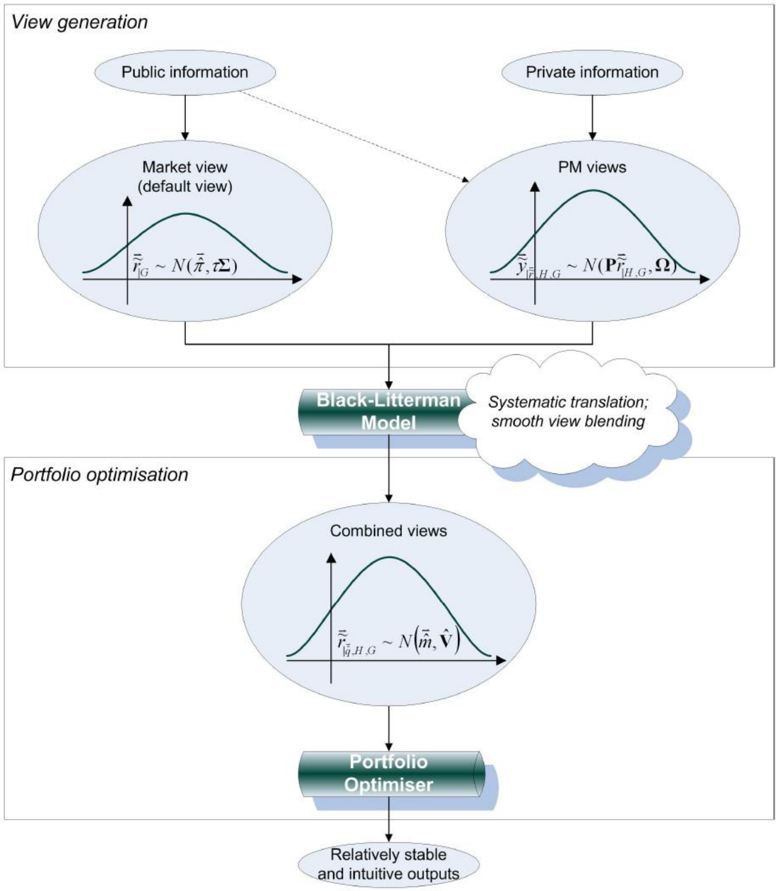
1	2	3
1	2	3
1	2	3

Black-Litterman (Goldman Sachs, 1992)

Expected returns are viewed as a blend of capital markets information and a set of investors' insights.

Investors should take the most risk where they have the strongest views.

The B-L model uses investors' views in the form of a Pick Matrix to complete the analysis.



Graphic: Wing Cheung, Nomura Securities, 2009

Pick Matrix

AAPL	MSFT	INTC	BAC	JPM	GE	JNJ	Return
1	-1	0	0	0	0	0	.08
0	0	0	1	-1	0	0	.03
0	0	-1	0	0	.5	.5	.04
0	0	0	0	0	1	0	.09

View 1: Apple will outperform Microsoft by 8% this year.
View 2: Bank of America will outperform JP Morgan by 3% this year.
View 3: The average of Johnson & Johnson and GE will outperform Intel by 4% this year.
View 4: GE will return 9% this year.

We then assign a confidence level to each view which calculates a range around each expected return. A low confidence creates a wide range, while a high confidence creates a narrow range.

Black-Litterman in Summary

Flexibility

- Active investors can incorporate their views on the market using Bayesian techniques. These views could be based on fundamental or technical analysis, for example.
- An investor who is market-neutral with respect to all securities should optimally hold the market.

Stability

Inputs such as views on the market, coupled with prior information, allow for more stable portfolios.

ESTIMATING VOLATILITY

Non-normality

- The assumption of normal distributions is not realistic for most asset returns.
- A Markov chain is simulated where the stationary distribution is the posterior (predictive) distribution.
- The output from the chain is a sample of nearly identical, but not independent, draws from the posterior distribution.

Volatility

There are two broad categories of volatility parameter changes:

- Structural breaks (permanent changes)

- Regime changes (e.g. business cycles, finite states of the world)

Volatility

- In regime-switching models, the state variable follows a discrete Markov chain.
- In the three regime case, volatility dynamics are governed by a matrix of transition probabilities.
- If time permits, we will describe the Brigo model for interest rate derivatives, outlined in the appendix.

References

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